9. Homework Assignment

Dynamical Systems II

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`http://dynamics.mi.fu-berlin.de/lectures/`

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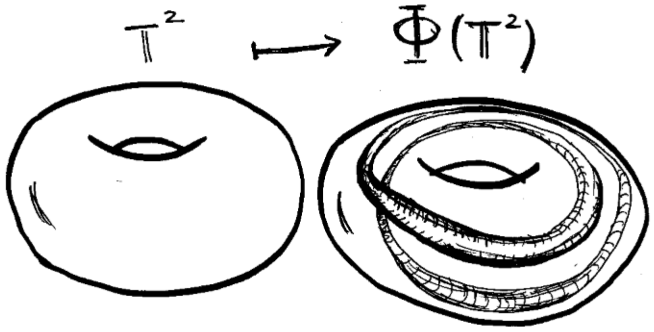
Problem 1: Consider a diffeomorphism $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ and a compact, Φ-invariant, hyperbolic set I.

Prove or disprove: the hyperbolic structure on I is unique.

Problem 2: [Smale Solenoid] Realize the following sketch by a continuously differentiable map Φ of the (solid) 2-Torus $\mathbb{T}^2 := \text{disk} \times S^1$ into itself.



(i) Give an example of Φ in suitable coordinates.

(ii) Prove that the attractor $\mathcal{A} := \bigcap_{n=0}^{\infty} \Phi^n(\mathbb{T}^2)$ of Φ is hyperbolic.

Hint: Consider the coordinates (x, y, φ) with $x^2 + y^2 \leq 1$ and $\varphi \in [0, 2\pi)$.

Problem 3: Consider the Plykin attractor as defined in class. It is obtained by the intersection of iterates of an initial domain $M = A \cup B \cup C \cup D$ under a diffeomorphism Φ,

$$P := \bigcap_{n=0}^{\infty} \Phi^n(M),$$

The domain M is compact, connected, and path-connected. The same holds true for all iterates $\Phi^n(M)$. Prove or disprove:

(i) The Plykin attractor is connected.

(ii) The Plykin attractor is path-connected.

Problem 4: A measure of complexity of a map Φ is the *topological entropy* h: Let $N(n)$ be the number of periodic points of Φ with (not necessarily minimal) period n. Then the entropy is defined as

$$h := \limsup_{n\to\infty} \frac{\log N(n)}{n}.$$

Calculate the entropy h of the shift on m symbols. Prove that every iteration Φ containing a shift (i.e. with an invariant set I such that $\Phi|_I$ is conjugate to a shift on m symbols) has positive topological entropy.